Regulating Non-point Pollution with Ambient Tax: Are more monitors better?

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Abstract

Non-point source pollution occurs when pollution is not observable at the individual level by the regulator. Traditional tools like Pigouvian taxes are not applicable here but a tax can be applied at the group level based on joint pollution production. Such policies are referred to as ambient taxes and they are a promising alternative to ameliorate the problem of non-point source pollution. Ambient taxes can induce polluters to collectively meet the pollution standard at least cost under certain settings. But in general, it may achieve the regulator's pollution target at higher than least cost if not at highest cost. This result is due to the presence of free-riding incentives. I show in this paper the conditions in which uncertainty about firm types may lead to incorrectly setting the uniform ambient tax rate which then creates the potential for free-riding. I also compare the Nash and Sub-game Perfect Nash equilibria and analyze the potential welfare gains of adding more water quality monitoring points. I find that expanding the network of water monitors in such a setting does not always reduce free riding potential and, in some cases, may increase it.

Introduction

Much of the environmental economics literature focuses on policies that are based on observable individual emissions such as Pigouvian taxes, tradable permits, and even Coasian bargaining mechanisms. However, there is an entire class of pollution problems that render individual emissions monitoring infeasible or prohibitively costly due to the sources of pollution being diffuse and/or the emissions transfer function being stochastic. Such occurrences are referred to as non-point source (NPS) pollution.

In the U.S., NPS pollution problems represent the last major hurdle to achieving water quality goals with agricultural runoff as the main source of such pollution (U.S. Environmental Protection Agency, 2016). Due to the unobservable nature of NPS pollution, traditional emissions-based policies cannot be applied. Fortunately, there are many alternatives in the policy toolbox from which to choose. The focus of this paper is on ambient taxes.

Basing policies on observable ambient quality can overcome many of the issues seen in other alternatives. First, it gives incentives to abate and it is flexible in regards to the method of abatement, unlike input taxes. Second, ambient-based policies are tied to observable pollution rather than estimates of it, like the emission proxies, making it more parsimonious and accurate. However, ambient-based policies have their own set of problems as well. One important issue is the potential for free-riding to occur since ambient-based policies are group-based incentives. The research goals in this paper are to (1) examine under what conditions free-riding can occur under both NE and SPNE when an ambient tax is imposed, and (2) does adding more monitoring points reduce the potential for free-riding?

Segerson (1988) and Meran and Schwalbe (1987) were the first to suggest ambient-based policies to correct for NPS pollution and showed how an ambient tax/subsidy can induce polluters to collectively choose the socially optimal level of ambient concentration as a Nash Equilibrium. However, their proposed solutions involve charging firm specific tax rates which require knowledge of firms' abatement cost functions thus placing an informationally expensive burden on the regulator. If knowledge about the distribution of types is available, it is possible to achieve first best abatement allocations with a uniform tax rate even with heterogeneous firms (Segerson and Wu, 2006). Without such information readily available, the regulator has two options. First, a damage-based tax can be implemented according to Hansen (1998) which only requires knowledge about the damage function at the socially optimal ambient level; however, even this can be argued as too informationally demanding. Alternatively, Segerson and Wu (2006) develop a regulatory threat mechanism whereby the regulator threatens the NPS polluters with an ambient tax if the polluters do not achieve the standard voluntarily. The ambient tax, if implemented, requires the regulator to invest resources to learn about the firm types in order to optimally set the tax rate. Unfortunately, this mechanism requires that the regulator's threat be credible so it is not a perfect workaround.

Any informational requirement about the damage function or distribution of firm types is, in practice, still a significant hindrance for the regulator even in a point-source pollution setting. This fact has motivated the environmental economics literature to shift its focus towards the second best policy solution which aims to achieve a set pollution target at least cost. The second best arises because of uncertainty about the optimal pollution target.

With NPS pollution however, an ambient tax is not guaranteed to achieve this second best target without the information requirements mentioned earlier. As noted in (Kotchen and Segerson, 2020), it is possible that an ambient tax can achieve its pollution target but with some degree of free-riding (i.e., some polluters discharge more than socially optimal). Thus, in the presence of free-riding, ambient taxes produce an outcome closer to a third best situation where the second best target is achieved but at greater than least cost. In this paper, I characterize the conditions under which free-riding can occur while achieving the pollution target and examine how expanding the monitoring network affects the degree of free-riding.

I find that when the uniform tax rate is set too high relative to the pollution target (i.e., ambient standard), then the possibility of achieving compliance at greater than least cost arises. This occurs because the collective tax gives rise to a minimum profitable pollution level ($\widetilde{X}_i$) for each polluter. This level of discharge is possibly heterogeneous and marks the lowest level of discharge that a firm is willing to commit towards avoiding the known tax penalty that would result from non-compliance. Intuitively, if a firm is pivotal in the determination of compliance, then they would not be willing to pollute below $\widetilde{X}_i$ to avoid the group tax penalty and would instead prefer to pollute at $\widetilde{X}_i$ and pay the tax rather than go below $\widetilde{X}_i$ to avoid the tax. Therefore, $\widetilde{X}_i$ is a decreasing function of the ambient tax rate.

This implies that there is a unique value for t such that the minimum profitable pollution level equals the least cost level for each polluter. Above this unique point for t, the minimum profitable pollution $\widetilde{X}_i$ is less than the least cost level X_i^*. When that happens, a multiplicity of Nash Equilibria (NEs) arises[1]. These NEs all achieve compliance (henceforth referred to as compliance NEs) but creates a situation where some polluters are polluting less than the least cost amount so that others can pollute more than the least cost level. Those who pollute above their least cost level are henceforth referred to as free-riders. This is the fundamental moral hazard problem prevalent in team games that lead to free-riding (Holmstrom, 1982).

The other main contribution of this paper is to examine the impact from adding more monitoring points on the degree of free-riding. The classic model from Segerson (1988) assumes that individual discharge monitoring is prohibitively costly but that group level monitoring is feasible. For example, having one monitoring point downstream of all known polluters. When one such monitoring point is feasible then it is natural to ask how many more monitors are feasible? The concept of feasibility here is determined by an implicit cost-benefit analysis. If the regulated body of water were a lake, such additional monitoring points would do little to change outcomes and thereby providing no real benefits. However, under a river network, adding more monitoring points could effectively partition the initial group of polluters into smaller groups, thereby creating less free-riding potential and possibly translating to real welfare benefits. Allowing the number of monitors to be greater than one is the more general model with the classic Segerson (1988) model as one special case in which the net-

[1] Least cost pollution level is the pollution level that a planner would choose (for the individual polluter) that results from a second best optimization problem.

work has only one monitoring point at the river's bottom. The other special case is when the number of monitors m equals the number of polluters n in which case the NPS pollution effectively becomes point source.

Such an analysis requires the examination of a particular outcome that can be examined as m increases. However, the presence of free-riding incentives only exists when there are a multiplicity of NEs. Thus, the Subgame Perfect Nash Equilibrium (SPNE) is used as an alternative concept where the player furthest upstream is the first mover and the player furthest downstream is the last mover. Using the SPNE has many advantages to the research goal. First, the SPNE is a refinement of the NE so it is itself an NE and remains unique regardless of the ambient tax rate. Secondly, the SPNE produces the highest cost compliance scenario when firms are homogeneous. Thus, even if there is no way to know which NE will occur, it is at least possible to examine how the highest cost NE would change with m. In this way, the SPNE outcome can be viewed as a measure of the maximum potential for free-riding.

Lastly, many water quality goals encompass rivers/streams which have a flow direction dictated by gravity allowing the possibility of differential "power" among polluters based on their river location. This idea has been studied in the literature on irrigation access as a public good (Bell et al., 2015; D'exelle and Lecoutere, 2012) and has recently been studied in a NPS pollution context (Zia et al., 2020; Miao et al., 2016). Both papers use experimental methods to find that as monitoring locations expand and frequency of measurement increases, upstream players' behavior is more affected relative to downstream players. However, in those models, the differential strategies between players located more upstream and those more downstream arises because of the differences in nutrient transport, specifically the process of nitrification. In this paper, such differences are ignored because the focus of this paper is on free-riding incentives which exist even without heterogeneity.

There are many other issues with ambient policies which are not addressed in this paper. For instance, efficacy of ambient policies requires firms to understand that their actions have effects on measurable ambient quality. This is typically only an issue when the number of polluters is large and thus ambient policies have much greater appeal in settings with few polluters. However, settings with few polluters could exacerbate the collusion issue that can arise under ambient policies (Cabe and Herriges, 1992). Collusion occurs when polluters have the ability to communicate with each other and find it collectively more profitable to over abate at the aggregate level. This paper abstracts away from the collusion issue by focusing on a pure ambient tax which gives no subsidies for over abatement. Combining this with the assumption of no stochasticity in the ambient quality buys us a model where there is no incentive to collude and over abate. This result is confirmed in the experimental literature on ambient policies for NPS pollution (Cochard, Willinger and Xepapadeas, 2005).

I begin with setting up the model for the social planner that tries to achieve an ambient goal at least cost. From this, a least cost policy is derived and used to benchmark against the other pollution outcomes discussed later. I then derive best response functions of firms in both the simultaneous and sequential game settings treating the policy as given and fixed. Then equilibria is discussed in both game types (simultaneous and sequential) and policy types (perfect tax and strict tax) along with the resulting welfare implications. Afterwards, I describe and model the potential for free-riding and examine whether location additional monitors matter and how increasing the number of monitors might impact the potential for free-riding.

Model Setup

Consider n known polluters (firms) whose discharge all go to the same monitoring point. Firms differ in location and are all situated along a simple linear river network and are allowed to be heterogeneous. I allow firms to freely choose their discharge level (X_i) and I abstract away from both output decisions and the consumer market. For now, I assume the regulator can only feasibly monitor ambient concentrations directly downstream of the last polluter, firm n. Later, I will allow the number of monitors to increase above one. Figure 4 depicts a two player example of the game tree when the game is assumed to be sequential.

The regulator observes ambient quality X, takes the ambient standard $\overline{X}$ as given and chooses a value for the uniform tax rate t that will induce a compliance outcome. Each firm faces an ambient tax given by (1)

$$T_i(X) = \begin{cases} 0 & \text{if } X \leq \overline{X} \\ t(X - \overline{X}) & \text{if } X > \overline{X} \end{cases} \tag{1}$$

and firm level payoffs are given by (2)

$$\Pi(X_i, X, \theta_i) = \begin{cases} \pi(X_i, \theta_i) & \text{if } X \leq \overline{X} \\ \pi(X_i, \theta_i) - t(X - \overline{X}) & \text{if } X > \overline{X} \end{cases} \tag{2}$$

where $\pi()$ is the farm profit for a chosen individual pollution level without tax burden considerations. The model setup will be general enough to allow for heterogeneity but I will invoke the homogeneity assumption to derive many of the results in this paper. Firm types are given by θ_i, the slope of the marginal profit of pollution curve (isomorphic to marginal abatement cost curve) and $\pi_i(X_i)$ is short hand for $\pi(X_i, \theta_i)$. A higher value for θ is assumed to have a positive effect on the marginal profit of pollution ($\frac{\partial^2 \pi_i}{\partial X_i \partial \theta_i} \geq 0$) and thus leading to a (weakly) higher level for X_i^{bau}, the business as usual level of discharge for each firm. The ambient pollution is assumed to be the sum of discharges across all polluters $X = \sum_{j=1}^{n} X_j$. Lastly, $\pi_i(\cdot)$ is assumed to be an upside down parabola where $\pi_i'(X_i^{bau}) = 0$ and $\pi(0) = 0$.

The Social Planner

The goal of this section is to pin down the optimal uniform tax rate. The planner wants to choose pollution allocations $(X_1, \ldots, X_n)$ so that ambient quality reaches the standard $(X \leq \overline{X})$ at least cost. The cost structure is subsumed in the farm profit function so that the planner's problem is framed as in (3).

$$\max_{\{X_i\}_{i=1}^{n}} \sum_{i=1}^{n} \pi(X_i, \theta_i) \qquad \text{s.t.} \qquad X \leq \overline{X} \tag{3}$$

From equation (3), the least cost pollution level for individual i, denoted as X_i^*, is pinned down by (4).

$$\pi'(X_i^*, \theta_i) - \lambda^*(\theta_1, \ldots, \theta_n, \overline{X}) \leq 0 \qquad \text{for } i = 1, \ldots, n \tag{4}$$

Thus the socially optimal pollution allocation is such that everyone pollutes up to the point where their marginal profit from pollution equals the shadow price of pollution given by the lagrange multiplier, $\lambda^*(\boldsymbol{\theta}, \overline{X})$. The least cost pollution allocation is then given by $(X_1^*, \ldots, X_n^*)$ and this achieves compliance exactly ($\sum X_i^* = \overline{X}$).

Firm Problem

Polluting firms face the following optimization problem

$$\max_{X_i} \Pi(X_i, X_{-i}, \theta_i) \tag{5}$$

where the objective function in (5) is given by (2). And since the tax schedule from (1) is piecewise, solving (5) requires analyzing the optimum on both sides of the kink (see Appendix for full solution).

Proposition 0.1. *For each player i, there exists a minimum profitable pollution level (denoted as $\widetilde{X}_i$) such that player i would never find it profitable to pollute below $\widetilde{X}_i$ to avoid the tax penalty given by $t(X_{-i} + \widetilde{X}_i - \overline{X})$ where X_{-i} denotes the pollution attributable to all but firm i. The minimum profitable pollution level is indirectly given by*

$$\pi'(\widetilde{X}_i, \theta_i) = t$$

Therefore we have $\widetilde{X}_i = \widetilde{X}_i(t)$ and $\widetilde{X}_i'(t) \leq 0$.

See proof on page 30.

Nash Equilibrium

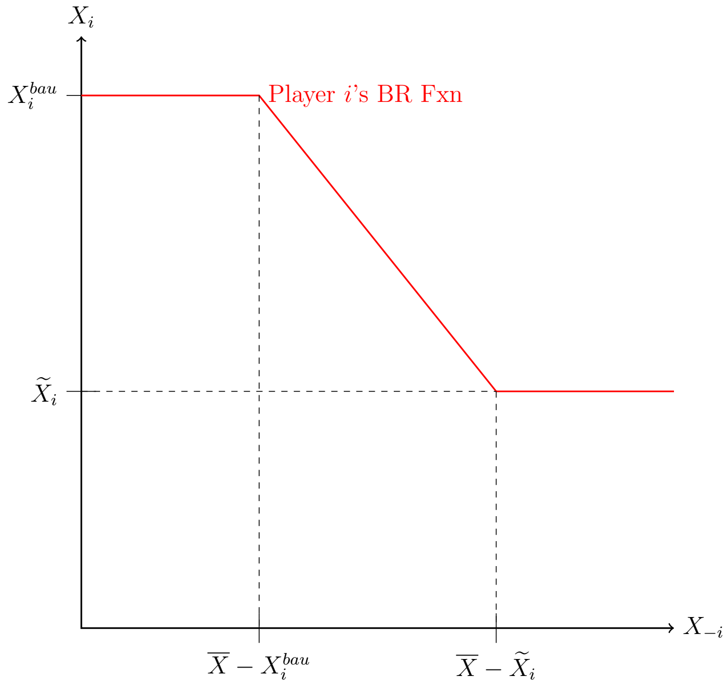
Here, I present the setup for the simultaneous game by deriving best response functions and the Nash Equilibrium. From Proposition 0.1, we know that no player would choose pollution below $\widetilde{X}_i$ no matter what. Furthermore, all players would like to choose X_i^{bau} if they could do so without incurring a penalty. Thus for all players, their best response function given the pollution level of others (denoted as X_{-i}) is given by (6).

$$X_i^{BR} = \begin{cases} X_i^{bau} & \text{if} \quad X_{-i} \leq \overline{X} - X_i^{bau} \\ \overline{X} - X_{-i} & \text{if} \quad \overline{X} - X_i^{bau} \leq X_{-i} \leq \overline{X} - \widetilde{X}_i \\ \widetilde{X}_i & \text{if} \quad X_{-i} \geq \overline{X} - \widetilde{X}_i \end{cases} \tag{6}$$

Equation (6) is depicted graphically in Figure 1. If player i knows that they can pollute business as usual without incurring the tax penalty then they will surely do so. However, if they cannot do so without being penalized, then they will cut back on pollution levels to avoid the penalty but only up to a certain point. All firms would rather contribute towards noncompliance rather than pollute below their minimum profitable pollution levels, $\widetilde{X}_i$. This result relies on the fact that polluters know exactly what their tax burden would be in the case of noncompliance and if uncertainty is allowed, firms will then need to know some moments of the distribution for the penalty.

From proposition 0.1 and equation (4), we see that if the regulator sets the uniform tax

Figure 1: Best Response Function for Pollution



rate so that it equals the lagrange multiplier λ^*, then all firms would have their minimum profitable pollution levels be equal to their least cost pollution levels and thus would result in a unique Nash Equilibrium where ambient quality target is met exactly and at least cost (Segerson and Wu, 2006).

However, assuming that the regulator knows the full distribution of $\boldsymbol{\theta}$ is not likely to hold in reality. Though it could be the case that the regulator can incur a cost to learn $\boldsymbol{\theta}$, this cost could easily be prohibitively high. When $\boldsymbol{\theta}$ is unknown, it invites the possibility of having a value $t \neq \lambda^*$ while still maintaining compliance; this occurs if $t > \lambda^*$.

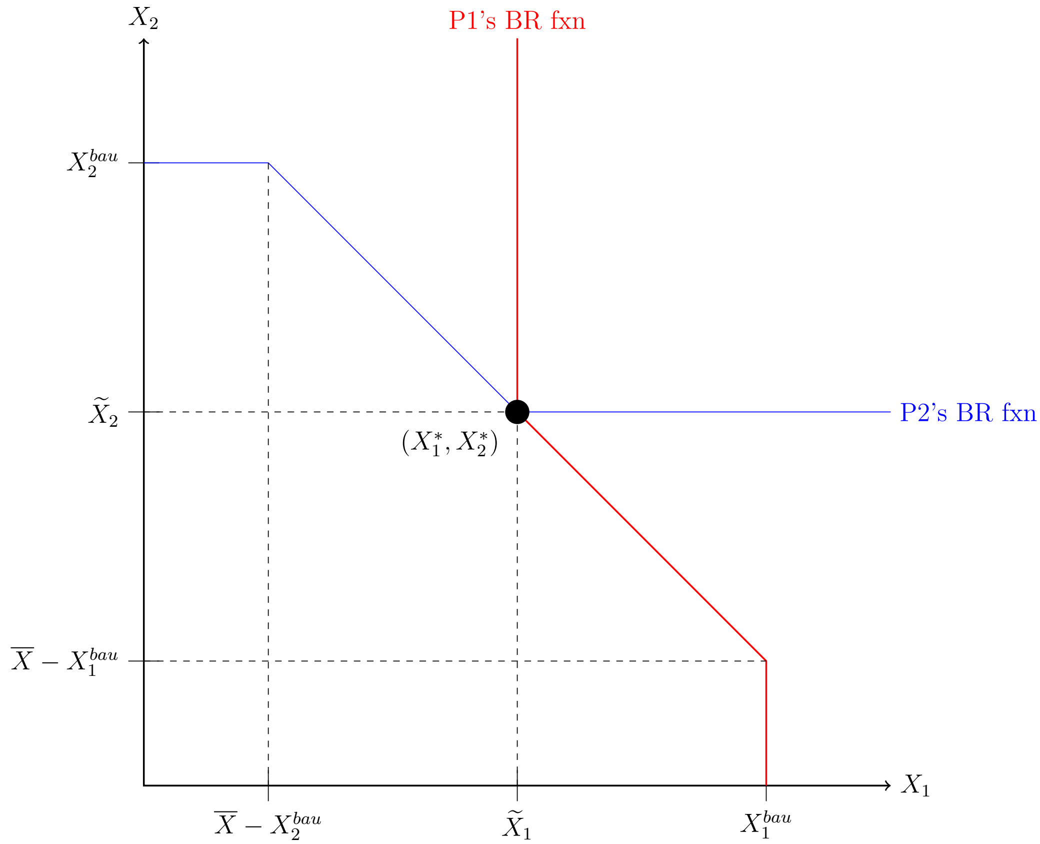
The Effects of t on Nash Equilibria

Here we look at the implications of various levels of t that would induce a compliance NE. From equation (4) and proposition 0.1, we know that when $t < \lambda^*$, then $\widetilde{X}_i > X_i^*$ for all i since we assume $\pi''() \leq 0$. This inevitably leads to noncompliance because no one is willing to pollute below their minimum profitable pollution levels which happens to be higher than the least cost pollution level for each i.

When $t = \lambda^*$, each player has their minimum profitable pollution level exactly equal to their least cost pollution level. When this happens, the only unique Nash Equilibrium pollution allocation occurs at the point where each firm pollutes exactly $\widetilde{X}_i$ as depicted in Figure 2. I refer to this value as the "perfectly" set tax rate. This choice of nomenclature captures the idea that when t is set equal to λ^*, players' best response functions intersect at exactly one point in the n-dimensional space. At that point, compliance is met perfectly and at least cost to polluters.

The novel result of this paper focuses attention on the case when t is set "too strictly" so that $t > \lambda^*$. This is shown in Figure 3 and when this occurs, all players' minimum profitable pollution level now lies below their least cost levels ($\widetilde{X}_i < X_i^*$). In such a setting, all firms are willing to pollute less than X_i^* but no less than $\widetilde{X}_i$ to avoid a tax penalty. Therefore, some firms can get away with polluting above X_i^* and free-ride off of those who are polluting less than what is optimal. This result is a consequence of having a multiplicity of NEs where any allocation outside of $\boldsymbol{X}^* = (X_1^*, \ldots, X_n^*)$ is an inefficient allocation even though all NE's

Figure 2: Two-Player Best Response Functions (perfectly set t)



achieve compliance. This is summarized in theorem 0.2 and the intuition for the result is very simple.

When $t > \lambda^*$, it creates a gap between $\widetilde{X}_i$ and X_i^*. This creates a potential surplus of implied pollution quotas available for some to pollute beyond X_i^*. The total potential surplus available to a firm i is given by (7). When homogeneity is assumed, equation (7) becomes $\overline{X} - n\widetilde{X}$.

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} \left(X_j^* - \widetilde{X}_j \right) \tag{7}$$

Theorem 0.2. *Let λ^* be the shadow price of pollution which is defined as in (4). When $t < \lambda^*$, then the NE is non-compliance such that*

$$X = \sum_{i=1}^{n} \widetilde{X}_i(t) > \overline{X}$$

But when $t = \lambda^*$, compliance is reached exactly (i.e., $\sum_{i=1}^{n} \widetilde{X}_i(t) = \overline{X}$). Lastly, when $t > \lambda^*$ we have

$$\sum_{i=1}^{n} \widetilde{X}_i(t) < \overline{X}$$

and the allocation $(\widetilde{X}_1, \ldots, \widetilde{X}_n)$ is no longer an NE. Instead, the NE is characterized by

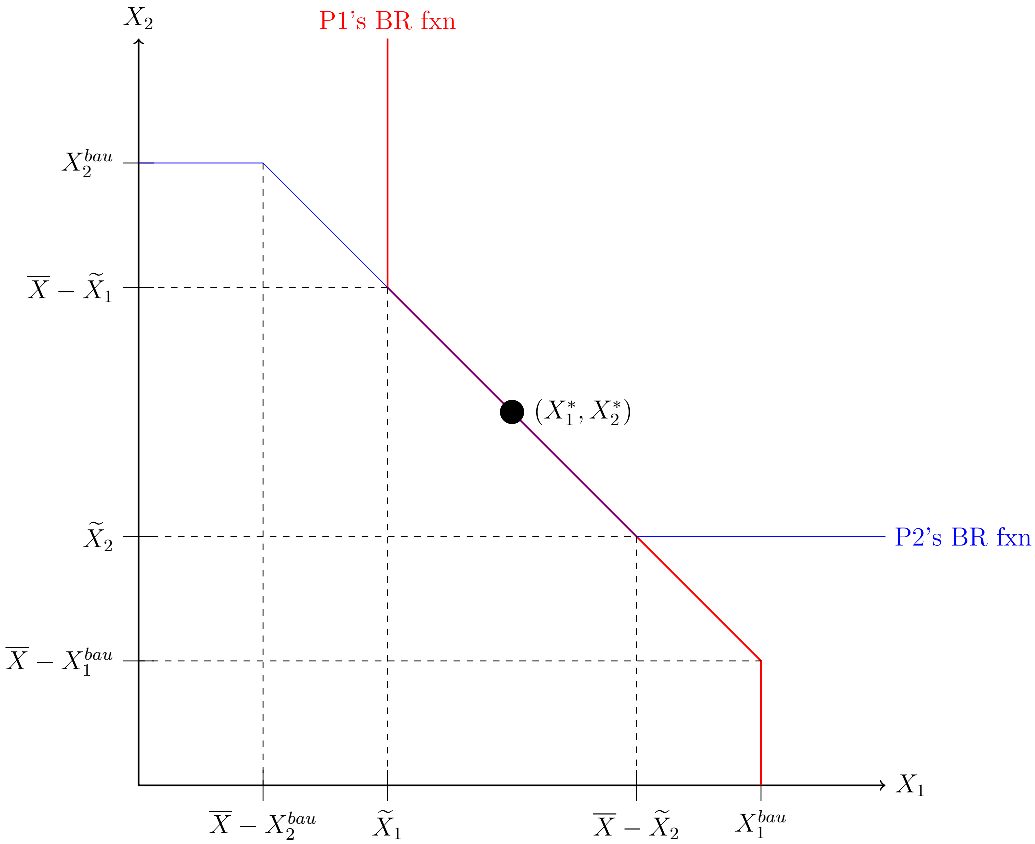
$$(X_1^{ne}, \ldots, X_n^{ne}) : \quad \sum_{i=1}^{n} X_i^{ne} = \overline{X}$$

with $X_i^{ne} \in \left[\widetilde{X}_i, X_i^{bau}\right]$ for all i. Therefore, the ability to free ride exists to the extent that $t > \lambda^*$.

See proof on page 32.

Without introducing any equilibrium selection concepts, there is no way to know which NE will be selected when $t > \lambda^*$. Worse yet, in the two player example with homogeneous types, the allocations $(X_1, X_2) = (\widetilde{X}_1, \overline{X} - \widetilde{X}_1)$ and $(X_1, X_2) = (\overline{X} - \widetilde{X}_2, \widetilde{X}_2)$ produces a compliance outcome at the highest cost. It turns out that under homogeneity and sequential play, then the Sub-game Perfect Nash Equilibrium will produce this "worst case scenario".

Figure 3: Two-Player Best Response Functions (too strict of t)

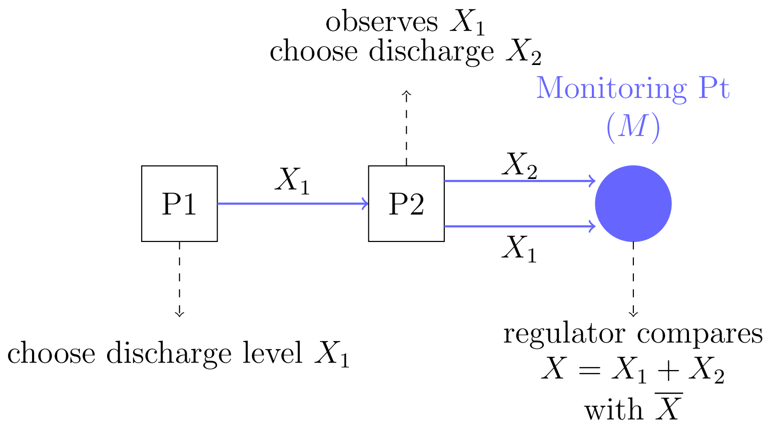


SPNE as "Worst Case"

It is helpful for expositional purposes to use the SPNE concept to characterize the "worst case scenario", that is, when compliance is reached at highest cost. Even though it may not be innocuous to assume that downstream players can perfectly observe the quality of water that reaches their own stretch, such an assumption allows us to examine the SPNE for other purposes. The extensive form game tree is depicted in Figure 4 assuming only two players. Player 1 is the first mover while player n is the last. Following standard procedure, we utilize backward induction and first pin down the last player's strategy.

The n^{th} player's strategy is exactly described by equation (6) since they are the last mover

Figure 4: 2 Player Example Model



in this game. Player $n-1$'s problem will slightly diverge from equation (6). Player $n-1$ still has the same value for their minimum profitable pollution level as in the simultaneous game. The difference here is that their compliance goal is not to stay under $\overline{X}$ since they are not the last person in the river. If player $n-1$'s goal is to reach compliance, then they must keep the ambient pollution discharged by everyone upstream of n (denoted as $X_{\uparrow(n)}$) to be weakly less than $\overline{X} - \widetilde{X}_n$ because if player $n-1$ pollutes too much so that $X_{\uparrow(n)}$ is too high, then player n will still choose $\widetilde{X}_n$ and thus push the group to be out of compliance. The best response function for player $n-1$ is then given by (8).

$$X_{n-1}^{BR} = \begin{cases} X_{n-1}^{bau} & \text{if} \quad X_{\uparrow(n-1)} \leq \overline{X} - X_{n-1}^{bau} - \widetilde{X}_n \\ \overline{X} - \widetilde{X}_n - X_{\uparrow(n-1)} & \text{if} \quad \overline{X} - X_{n-1}^{bau} - \widetilde{X}_n \leq X_{\uparrow(n-1)} \leq \overline{X} - \widetilde{X}_n - \widetilde{X}_{n-1} \\ \widetilde{X}_{n-1} & \text{if} \quad X_{\uparrow(n-1)} \geq \overline{X} - \widetilde{X}_n - \widetilde{X}_{n-1} \end{cases} \tag{8}$$

The term $X_{\uparrow(n-1)}$ denotes the pollution amount attributable to polluters upstream of $n-1$ and is not to be confused with $X_{-(n-1)}$ which is the pollution amount attributable to all polluters excluding $n-1$. Equation (8) essentially translates equation (6) to the sequential context so that now player $n-1$ takes only upstream pollution ($X_{\uparrow(n-1)}$) as given and their compliance goal explicitly takes the next player's strategy into account.

Player $n-1$ knows that player n will never choose to pollute below their minimum profitable pollution level ($\widetilde{X}_n$). Therefore, if player $n-1$ wants to achieve a compliance outcome, she must ensure that the water received by n does not exceed $\overline{X} - \widetilde{X}_n$. However, if the water received by $n-1$ is too polluted so that $n-1$ must pollute below her own minimum profitable pollution level to achieve the compliance goal, then she will surely not do so resulting in overall non-compliance. Equation (9) extends the best response function to all players j where j is upstream of n ($j < n$). For (9) to apply to Player 1, simply set $X_{\uparrow(1)} = 0$.

$$X_j^{BR} = \begin{cases} X_j^{bau} & \text{if} \quad X_{\uparrow(j)} \leq \overline{X} - X_j^{bau} - \sum_{k=j+1}^{n} \widetilde{X}_k \\ \overline{X} - X_{\uparrow(j)} - \sum_{k=j+1}^{n} \widetilde{X}_k & \text{if} \quad \overline{X} - X_j^{bau} - \sum_{k=j+1}^{n} \widetilde{X}_k \leq X_{\uparrow(j)} \leq \overline{X} - \widetilde{X}_j - \sum_{k=j+1}^{n} \widetilde{X}_k \\ \widetilde{X}_j & \text{if} \quad X_{\uparrow(j)} \geq \overline{X} - \widetilde{X}_j - \sum_{k=j+1}^{n} \widetilde{X}_k \end{cases} \tag{9}$$

Welfare in Equilibrium

There is a well known result that the SPNE is a subset of the set of NE's and so when t is set perfectly, there is a unique NE and is thus identical to the SPNE. When t is set too strictly (high), however, the SPNE results in one of the extremes from the set of NE's. Take Figure 3 as an example. Assuming that Player 1 is upstream of 2, the SPNE would produce the point $(\overline{X} - \widetilde{X}_2, \widetilde{X_2})$ as the pollution allocation[2]. This is because Player 1 can exert its first mover advantage over Player 2 by producing more pollution and forcing Player 2 to pick up the slack. Player 2 is happy to do this because of Proposition 0.1. This intuition is captured in Corollary 0.3 and Theorem 0.4.

Corollary 0.3. *If all firms are identical in all but location, then the SPNE resulting from the policy $(t, \overline{X})$ where $t > \lambda^*(\boldsymbol{\theta}, \overline{X})$ produces a pollution allocation that is non-increasing downstream. That is*

$$X_h^{spne} \geq X_\ell^{spne}$$

where $h < \ell$ (so that h is upstream of ℓ).

See proof on page 32.

Theorem 0.4. *Suppose all firms are homogeneous except location and that $t > \lambda^*$. Define $k = \lfloor \widetilde{k} \rfloor > 0$ where $\widetilde{k} = max\{B\}$ and $B = \left\{ \overline{k} \in \mathbb{R}_0^+ : \ \overline{k} X^{bau} + (n - \overline{k})\widetilde{X} = \overline{X} \right\}$. Then the SPNE would produce the following result*

$$(X_1^{spne}, \ldots, X_k^{spne}, \ldots, X_n^{spne}) = (\underbrace{X^{bau}, \ldots, X^{bau}}_{k}, X_R, \underbrace{\widetilde{X}, \ldots, \widetilde{X}}_{n-k-1})$$

and compliance is met exactly so that

$$\overline{X} = kX^{bau} + (n - k - 1)\widetilde{X} + X_R$$

[2]Assuming that $\overline{X} - \widetilde{X}_2 < X_1^{bau}$

where $X_R \in \left(\widetilde{X}, X^{bau}\right)$ *and* $k = \left\lfloor \dfrac{\overline{X} - n\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor$.

See proof on page 33. Note that $\lfloor \quad \rfloor$ is the floor function/operator.

Theorem 0.4 simply says the SPNE would be so that the first k players will choose BAU levels, player $k + 1$ will pollute some amount between BAU and minimum profitable level (henceforth referred to as the residual polluter), and the remaining downstream players pollute the minimum profitable pollution level. This formalizes the arguments made in the previous section and shows that the SPNE does indeed produce the most extreme allocation possible when $t > \lambda^*$.

The result from theorem 0.4 above and theorem 0.5 below implies that the SPNE still achieves compliance but at the highest possible cost. This result relies heavily on the homogeneous firm assumption which limits its value somewhat.

Theorem 0.5. *The SPNE allocation in Corollary 0.4 produces the lowest welfare possible among all other compliance NE's.*

See proof on page 33.

When we start to consider the more realistic scenario where firms are heterogeneous, the welfare consequences of a too strict t value in an SPNE becomes more complicated. In particular, the SPNE no longer coincides with the "worst case scenario" which is occurs when the lowest θ types are free-riding off of the highest θ types. In other words, the "worst case" occurs when those who stand to gain the least from free-riding, free-ride off of those who stand to gain the most from free-riding.

Optimal Location of Additional Monitors

The regulatory structure of applying an ambient tax on NPS polluters along a river will change as m, the number of monitoring points, changes. How that changes may depend on where those monitors are located. Towards that end, I will invoke the assumption that firms are homogeneous in all but location and this includes each firm's marginal damage of pollution. When an additional monitor is placed upstream of player $\ell + 1$, then it would effectively split the group of n polluters into two groups; the upstream group would have size ℓ while the downstream group has size $n - \ell$.

It is assumed that the regulator only cares about total pollution downstream of player n so that the resulting two groups are regulated independently. Since homogeneity is assumed here, the ambient tax rate that is applied to each section are the same. The ambient standard for each section will depend on where the second monitor is located. The ambient standard for the upstream section is

$$\overline{X}_u = \frac{\ell}{n}\overline{X}$$

and the standard for the downstream section is

$$\overline{X}_d = \frac{n-\ell}{n}\overline{X}$$

so that the sum of the two standards equals the original standard for the case $m = 1$. It should be noted that the process of determining the appropriate standard for each section is extremely simplified here where each section gets a representative share of the total original standard. Under homogeneity, this arbitrary process happens to be the optimal choice for the regulator since $X_i^* = \frac{\overline{X}}{n}$ for all i. But when heterogeneity is allowed, this simple process no longer corresponds to the optimal standard for each section. The corresponding total profits for both upstream (subscript u) and downstream (subscript d) groups are given in (10).

$$\begin{aligned} \Pi_u &= k_u \pi(X^{bau}) + (\ell - k_u - 1)\pi(\widetilde{X}) + \pi(X_{R_u}) \\ \Pi_d &= k_d \pi(X^{bau}) + (\ell - k_d - 1)\pi(\widetilde{X}) + \pi(X_{R_d}) \end{aligned} \tag{10}$$

The values (k_h, X_{R_h}) are defined similarly to (k, X_R) from theorem 0.4 but are derived from $\overline{X}_h$ for $h = \{u, d\}$. Thus welfare is given by (11).

$$W = \Pi_u + \Pi_d - D(\overline{X}) \tag{11}$$

Equations (10) and (11) indicate that the choice in location (ℓ) affects welfare through two channels: (1) how it affects total number BAU polluters $(k_u + k_d)$ and how it affects the amount that the two residual polluters discharge (X_{R_u} and X_{R_d}).

Theorem 0.6. *Initially let $m = 1$. Under homogeneity, the welfare maximizing location for the second monitor is $\ell = \frac{n}{2}$.*

See proof on page 34.

Theorem 0.6 suggests that the optimal location for the additional $m - 1$ monitors would be so that the entire river is partitioned evenly. Without a formal proof of this, for now I treat this as more of a simplifying assumption.

Optimal Number of Monitors

Now that it is established where the additional $m-1$ monitors will be located, the question that remains is how should m be determined so that the potential for free-riding is minimized? There will be m sections that are partitioned along the river (the number of sections equals the number of monitors) and homogeneity means that each section is regulated the same and thus behaves the same (has the same pollution outcome). The welfare under an m monitor regime is given by equation (12).

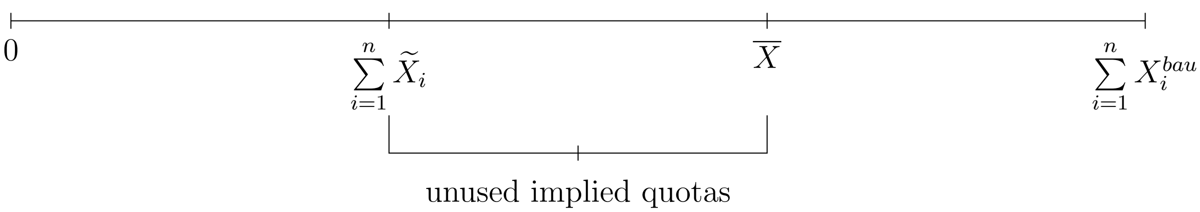
$$W_m = m\left[k_m\pi(X^{bau}) + (n_m - k_m - 1)\pi(\widetilde{X}) + \pi(X_{R_m})\right] - D(\overline{X}) \tag{12}$$

The bracketed term is the profits to each section so that (k_m, n_m, X_{R_m}) are respectively, the number of BAU polluters, number of total polluters, and the discharge level of the residual polluter for each identical section or group. By construction, $n_m = \frac{n}{m}$ and it is assumed that it is an integer. Further, the number of BAU polluters for each section is given by (13).

$$\begin{aligned}
k_m &= \left\lfloor \widetilde{k}_m \right\rfloor \\
\widetilde{k}_m &= max\{B_m\} \\
\widetilde{k}_m &= \frac{\overline{X} - n\widetilde{X}}{m(X^{bau} - \widetilde{X})} = \frac{\widetilde{k}}{m} \\
B_m &= \left\{\overline{k}_m \in \mathbb{R}_0^+ :\ \overline{k}_m X^{bau} + (n_m - \overline{k}_m)\widetilde{X} = \overline{X}_m\right\}
\end{aligned} \tag{13}$$

The number of BAU polluters per section (weakly) decreases with m but the number of sections increases with m. This begs the question of whether increasing the number of evenly space monitors actually affects the potential for free-riding in the manner that is socially desirable. The potential for free-riding is measured by the sum of all individual differences between the least cost pollution level and their minimum profitable pollution level. This is stated in the definition of k from theorem 0.4 when you recognize that the least cost pollution level for each polluter equals $\frac{\overline{X}}{n}$ under homogeneity. Figure 5 depicts the intuition for where the potential for free-riding comes from in a more general way that allows for heterogeneity.

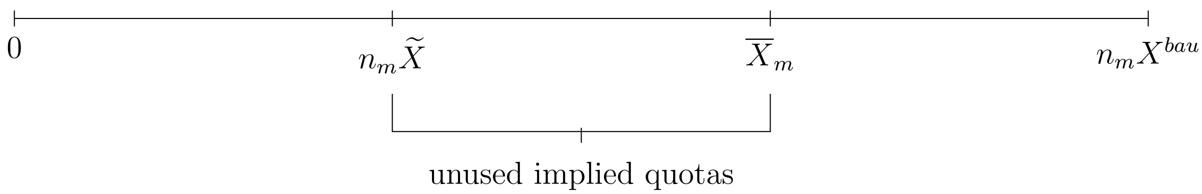
Figure 5: Free-Riding Potential, $m = 1$ and Heterogeneity Case



As a starting point, consider the case in which everyone is producing at their minimum profitable levels, $\widetilde{X}_i$. This is not an NE because there are unused implied pollution quotas left, i.e., $\sum_{i=1}^{n} \widetilde{X}_i < \overline{X}$; specifically, there would be $\overline{X} - \sum_{i=1}^{n} \widetilde{X}_i$ many unused implied quotas. How many BAU polluters that this unused implied quotas can support depends on the distance between X_i^{bau} and $\widetilde{X}_i$ for each i. Therefore, under homogeneity we see that theorem 0.4 defines the maximum BAU polluters for a compliance NE to be $k = \left\lfloor \frac{n\left(\frac{\overline{X}}{n} - \widetilde{X}\right)}{X^{bau} - \widetilde{X}} \right\rfloor$.

When we allow for $m > 1$, then the free-riding potential within each section depends on the distance between $\overline{X}_m$ and $n_m\widetilde{X}$. Since $\overline{X}_m$ is decreasing faster with m than $n_m\widetilde{X}_m$ does, then the free-riding potential within a section decreases as well.

Figure 6: Free-Riding Potential, $m > 1$ and Homogeneity Case



The total number of BAU polluters for the entire river, denoted as k_m^T is given by (14). At a glance, it is unclear whether total free-riding potential measured by (14) increases or decreases with m. Certainly when $m = n$, the potential for free-riding vanishes which would seem to indicate that the total free-riding potential will decrease with the number of monitors. However, this is not the case and I show how that works formally below and try to provide intuition along the way.

$$k_m^T = mk_m \tag{14}$$

Equation (14) implies that

$$k_1^T = \left\lfloor \frac{\overline{X}_1 - n_1\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor \tag{15}$$

where $\overline{X} = \overline{X}_1$ and $n = n_1$. Intuitively, when $m = 1$ the resulting number of total BAU polluters is at its maximum ($k_1^T = \max_m\{k_m\}$) and when $m = n$ then that number is now at its minimum ($k_n^T = \min_m\{k_m\}$). But theorem 0.7 shows that the function k_m^T does not in generaly change in a monotone fashion. Instead, there is a condition for which k_m^T actually increases with m, though never above k_1^T as stated in corollary 0.8.

Theorem 0.7. *Let k_m and k_m^T be the maximum number of BAU polluters within a section* *and the total maximum BAU polluters along the river, respectively. Then for $m_1 < m_2$ we have that $k^T_{m_1} > k^T_{m_2}$ if and only if equation (16) holds.*

$$\frac{k_1 - k_2}{k_1} > \frac{m_2 - m_1}{m_2} \tag{16}$$

See proof on page 36.

Corollary 0.8. *Increasing m may increase k^T_m but never more than k^T_1. In other words, a maximum (not necessarily unique) for k^T_m is k^T_1.*

See proof on page 37.

In general, equation (16) is not guaranteed to hold which means that the total maximum number of BAU polluters along the entire river, is not guaranteed to decrease with increases in the number of evenly spaced monitors. Its important to clarify that we are referring to the maximum possible number of BAU polluters that could result from a simultaneous game since there are so many possible NEs when $t > \lambda^*$. Under a more sequential type game, which is more plausible in a river setting as opposed to a lake, the result from theorem 0.7 is a proper prediction of a behavioral outcome.

Intuitively, theorem 0.7 says that an increase in monitors from m_1 to m_2 would decrease the potential for free-riding *for the entire river* if and only if the percent decrease in the potential for free-riding *within a section*, is greater than the percent increase in monitors (i.e., number of sections) relative to the new value, m_2. Restating this graphically, turn to figure 6. The result from theorem 0.7 means that the distance between the values $\overline{X}_m$ and $n_m\widetilde{X}$ must decrease faster than m is increasing in order for an increase in m to decrease the total potential to free-ride[3].

Discussion

The results suggest that when the regulator can calibrate t perfectly for $\overline{X}$, then there is no difference in the NE and SPNE results. However, if the regulator overshoots λ^* even slightly, then the NE and SPNE diverge. For the case in which polluters are homogeneous in all but location, the SPNE will always produce the worst compliance outcome in terms of welfare. This is because if t is set too strict (i.e., $t > \lambda^*$), then firms' minimum profitable pollution level ($\widetilde{X}_i$) is lower than the individual least cost discharge, X_i^*. This gap gives rise to the potential for free-riding leading to a wide range of pollution allocations that can both achieve compliance exactly and is individually rational. But when $t = \lambda^*$, firms' minimum profitable pollution level is exactly equal to the socially optimal individual discharge. This situation leads to no potential for free-riding and thus a unique NE would result.

Under homogeneity, the SPNE outcome being the highest cost compliance NE serves as a useful measure for the potential for free-riding under a simultaneous game. The main contribution of this shows that when the potential for free-riding exists, additional monitoring points may not necessarily decrease that potential. The degree to which additional monitors decreases that potential, depends on the extent in which the relative decrease in the potential for free-riding within each section is higher than the relative increase in the number of sections.

[3] To see that the distance between $\overline{X}_m$ and $n_m\widetilde{X}$ does decrease with m, simply take the derivative of both values with respect to m and compare.

Advances in water quality monitory technology has made this line of questioning more relevant than ever, reducing the cost of implementing more complex monitoring networks. Understanding the benefits of additional monitoring locations is thus a crucial component analysis in determining the benefits of such an investment. The policy implication of this paper's results is that if a regulator seeks to achieve a NPS pollution goal in a river-like system through the implementation of an ambient tax, they can do so but at an unknown cost a priori. The regulator can increase the number of monitoring points, but doing so does not guarantee that the resulting outcome is closer to the least-cost outcome. However, additional monitoring locations is more likely to achieve an outcome closer to the least-cost outcome when the level increase in the number of monitors is large as is apparent from equation 16.

Lastly, our theoretical results are consistent with the laboratory results from Miao et al. (2016) and Zia et al. (2020) though for different reasons. Those studies sought to examine the effects on polluter behavior under an ambient tax of changes in the information structure such as additional monitoring points or increase in monitoring frequency. This paper addresses the former directly providing a proper theoretical framework for that line of analysis. However, those two papers only went as far as one additional monitoring point which limits the effect on behavior somewhat.

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Appendix: Proofs

Proof of Proposition 0.1. We can rewrite 5 as

$$\max_{X_i^c, X_i^d} \left\{ \Pi_i^c, \Pi_i^d \right\} \tag{17}$$

where

$$X_i^c = \arg\max_{X_i} \ \pi_i(X_i) \qquad \text{s.t} \qquad X \leq \overline{X} \tag{14.1}$$

$$\Pi_i^c = \pi_i(X_i^c) \tag{14.2}$$

$$X_i^d = \arg\max_{X_i} \ \pi_i(X_i) - t(X - \overline{X}) \qquad \text{s.t} \qquad X > \overline{X} \tag{14.3}$$

$$\Pi_i^d = \pi_i(X_i^d) - t(X_i^d + X_{-i} - \overline{X}) \tag{14.4}$$

$$\widetilde{X}_i : \pi_i'(\widetilde{X}_i) = t \tag{14.5}$$

Here, X_{-i} denotes the pollution level of all other players but i. The strategy of player i can be broken down into two types, a comply strategy and a don't comply strategy. The comply strategy and associated payoffs are represented with a superscript c as in (14.1)-(14.2). The don't comply strategy and corresponding payoff is represented with a d as in (14.3)-(14.4).

Recognize that if $\widetilde{X}_i + X_{-i} \leq \overline{X}$ then $X_i^c \geq \widetilde{X}_i$ by definition. We want to show that $\Pi_i^c \geq \Pi_i^d$ if and only if $\widetilde{X}_i + X_{-i} \leq \overline{X}$. I prove this below.

Suppose that $\Pi_i^c \geq \Pi_i^d$.

$$\iff \pi(X_i^c) \geq \pi(X_i^d) - t(X - \overline{X})$$

$$\iff t(X - \overline{X}) \geq \pi(X_i^d) - \pi(X_i^c)$$

$$\iff t(X_i^d - X_i^c) \geq \pi(X_i^d) - \pi(X_i^c) \qquad \text{(add/subtract by } t(X_i^c) \text{ to LHS)}$$

$$\iff t \geq \frac{\pi(X_i^d) - \pi(X_i^c)}{X_i^d - X_i^c}$$

Note that there are only two possibilities, either the "don't comply" (DC) constraint binds $(\widetilde{X}_i + X_{-i} \leq \overline{X})$ or it doesn't $(\widetilde{X}_i + X_{-i} > \overline{X})$. When the DC constraint binds, then we have $\widetilde{X}_i \leq X_i^c < X_i^d$. When it fails to bind, we have $X_i^c < X_i^d = \widetilde{X}_i$.

Since we know that $t = \pi'(\widetilde{X}_i)$, then if $t \geq \dfrac{\pi(X_i^d) - \pi(X_i^c)}{X_i^d - X_i^c}$ holds, it must be the case that the DC constraint binds since both X_i^c and X_i^d are to the right of $\widetilde{X}_i$. When the DC constraint fails to bind, (i.e., $X_i^d = \widetilde{X}_i$ and $X_i^c < \widetilde{X}_i$), then the slope condition will fail to hold also. Thus

$$\Pi_i^c \geq \Pi_i^d \iff \widetilde{X}_i + X_{-i} \leq \overline{X} \qquad \text{(DC constraint binds)}$$

Therefore, when X_{-i} is small enough so that firm i can pollute at a minimum of $\widetilde{X}_i$ and still achieve $\overline{X}$ then it will do so. However, when X_{-i} is sufficiently large so that when firm i

chooses $\widetilde{X}_i$ it would not be compliant, then firm i will still choose to pollute $\widetilde{X}_i$. $\square$

Proof of Theorem 0.2. By definition we have

1. $\sum_{i=1}^{n} \widetilde{X}_i(\lambda^*) = \overline{X}$

   or equivalently we can state $\widetilde{X}_i(\lambda^*) = X_i^*$

2. $\widetilde{X}_i(t) = \left(\frac{\partial \pi_i(X_i)}{\partial X_i}\right)^{-1}(t)$

3. $\pi_i'(X_i)$ is convex and weakly decreasing over $\left[0, X_i^{bau}\right]$

By point two and three above, $\widetilde{X}_i(t)$ is also convex and decreasing in t. Therefore, when $t < \lambda^*$ the first point above implies that $\sum_{i=1}^{n} \widetilde{X}_i > \overline{X}$. It also proves the case for $t = \lambda^*$. And when $t > \lambda^*$ we have that $\widetilde{X}_i < X_i^*$. $\square$

Proof of Corollary 0.3. The proof follows directly from Equation (9). For example, take the first line from the piecewise function and compare this for two different players, h and ℓ where h is upstream from ℓ. Player h will pollute X^{bau} if $X_{\uparrow(h)} \leq \overline{X} - X_h^{bau} - (n-h)\widetilde{X}$ and Player ℓ will also pollute X^{bau} if $X_{\uparrow(\ell)} \leq \overline{X} - X_\ell^{bau} - (n-\ell)\widetilde{X}$. Since we have

$$X_{\uparrow(\ell)} > X_{\uparrow(h)}$$

and

$$\overline{X} - X_\ell^{bau} - (n-\ell)\widetilde{X} > \overline{X} - X_h^{bau} - (n-h)\widetilde{X}$$

it is hard to tell which player is more likely to play their BAU levels at the moment. However, we can establish that

$$X_{\uparrow(\ell)} - X_{\uparrow(h)} > (\ell - h)\widetilde{X}$$

since players in between h and ℓ would, at a minimum, produce $\widetilde{X}$. This then allows us to claim that the condition for h to play their BAU level is more likely to hold than the condition for ℓ to play their BAU level. A similar process can be done for the remaining pieces from (9) to fully establish the proof. □

Proof of Theorem 0.4. The SPNE pollution allocation follows directly from the definition of k and equation (8). To see that $X_R \in \left(\widetilde{X}, X^{bau}\right)$, notice that we have

$$\Longrightarrow \widetilde{k}X^{bau} + (n - \widetilde{k})\widetilde{X} = kX^{bau} + (n - k - 1)\widetilde{X} + X_R$$

$$\Longrightarrow (\widetilde{k} - k)X^{bau} + \left(1 - \left[\widetilde{k} - k\right]\right)\widetilde{X} = X_R$$

$$\Longrightarrow \sigma X^{bau} + (1 - \sigma)\widetilde{X} = X_R \qquad \text{(where } \sigma \in (0,1))$$

□

Proof of Theorem 0.5. The welfare from Proposition 0.5 is given by

$$W_0 = k\pi(X^{bau}) - (n - k - 1)\pi(\widetilde{X}) + \pi(X_R) - D(\overline{X})$$

but since all NE's result in compliance, the damage function will be same when comparing across different NE's and there is no tax incurred. Thus, the only relevant comparison is done on producer welfare, W_0^p given by

$$W_0^p = k\pi(X^{bau}) - (n-k-1)\pi(\widetilde{X}) + \pi(X_R)$$

The only possible reallocation will be one in which a free rider will pollute ε less while a contributor will pollute ε more. Such a reallocation produces welfare W_1^p where

$$W_1^p = (k-1)\pi(X^{bau}) + (n-k-1)\pi(\widetilde{X}) + \pi(X^{bau} - \varepsilon) + \pi(X_R + \varepsilon)$$

Since it would increase welfare marginally more if we give the ε to a polluter at the level of $\widetilde{X}$ over one at X_R. Then evaluating $W_0^p - W_1^p$

$$\begin{aligned} W_0^p - W_1^p &= \pi(X^{bau}) - \pi(X^{bau} - \varepsilon) + \pi(X_R) - \pi(X_R + \varepsilon) \\ &= \pi(X^{bau}) - \pi(X^{bau} - \varepsilon) - \Big[\pi(X_R + \varepsilon) - \pi(X_R)\Big] \\ &\leq 0 \qquad\qquad \text{(since } \pi() \text{ is concave or } \pi'(x) \text{ is decreasing in } x \text{ for } x \leq X^{bau}) \end{aligned}$$

□

Proof of Theorem 0.6. First I show that the choice of ℓ doesn't change the value $k_u + k_d$ and then I show that the choice $\ell = \dfrac{n}{2}$ maximizes $\pi(X_{R_u}) + \pi(X_{R_d})$.

By definition we have $k_u + k_d = \lfloor \widetilde{k}_u \rfloor + \lfloor \widetilde{k}_d \rfloor$. We can decompose $\widetilde{k}_h$ for $h \in \{u, d\}$ as

$$\widetilde{k}_h = \lfloor \widetilde{k}_h \rfloor + b_h$$

where $b_h \in (0, 1)$. Then we have

$$\lfloor \widetilde{k}_u + \widetilde{k}_d \rfloor = \lfloor \widetilde{k}_u \rfloor + \lfloor \widetilde{k}_d \rfloor + \lfloor b_u + b_d \rfloor$$

and since

$$\lfloor b_u + b_d \rfloor = \begin{cases} 0 & \text{if} \quad b_u + b_d < 1 \\ 1 & \text{if} \quad b_u + b_d \geq 1 \end{cases}$$

then

$$\lfloor \widetilde{k}_u \rfloor + \lfloor \widetilde{k}_d \rfloor = \begin{cases} \lfloor \widetilde{k}_u + \widetilde{k}_d \rfloor & \text{if} \quad b_u + b_d < 1 \\ \lfloor \widetilde{k}_u + \widetilde{k}_d \rfloor - 1 & \text{if} \quad b_u + b_d \geq 1 \end{cases}$$

Now that we have this expression, we see that

$$\arg\min_{\ell} \lfloor \widetilde{k}_u \rfloor + \lfloor \widetilde{k}_d \rfloor = \arg\min_{\ell} \lfloor \widetilde{k}_u + \widetilde{k}_d \rfloor$$

But since

$$\lfloor \widetilde{k}_u + \widetilde{k}_d \rfloor = \left\lfloor \frac{\frac{\ell}{n}\overline{X} - \ell\widetilde{X}}{X^{bau} - \widetilde{X}} + \frac{\frac{n-\ell}{n}\overline{X} - (n-\ell)\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor = \left\lfloor \frac{\overline{X} - n\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor$$

Thus, the location of the second monitor does not change $k_u + k_d$.

The only channel in which location affects welfare then is through the sum

$$\pi(X_{R_u}) + \pi(X_{R_d})$$

Denote ℓ^* as the location that maximizes the above sum. Suppose that ℓ^* is such that $X_{R_u} \neq X_{R_d}$. Then this contradicts the definition of ℓ^* because by shifting some pollution from the higher X_R to the lower X_R it would increase the join profit since $\pi()$ is concave. Thus, the necessary condition for ℓ^* to be the argmax of the joint profits is that $\ell^* : \; X_{R_u} = X_{R_d}$.

By definition we have that

$$X_{R_u} = \frac{\ell}{n}\overline{X} - \left\lfloor \frac{\frac{\ell}{n}\overline{X} - \ell\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor X^{bau} - \left(\ell - \left\lfloor \frac{\frac{\ell}{n}\overline{X} - \ell\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor - 1\right)\widetilde{X}$$

$$X_{R_d} = \frac{n-\ell}{n}\overline{X} - \left\lfloor \frac{\frac{n-\ell}{n}\overline{X} - (n-\ell)\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor X^{bau} - \left(n - \ell - \left\lfloor \frac{\frac{n-\ell}{n}\overline{X} - (n-\ell)\widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor - 1\right)\widetilde{X}$$

Therefore, the only ℓ that satisfies the necessary condition for maximization is $\ell = \dfrac{n}{2}$. By equations 10 and 11, this is also the location that maximizes total welfare. $\square$

Proof of Theorem 0.7. Let $m_1 < m_2$. We seek to find the conditions under which $k^T_{m_1} > k^T_{m_2}$.

Suppose that it's true and let $m_1 = m_2 - c$.

$$k^T_{m_1} > k^T_{m_2}$$

$$\iff m_1 k_1 > m_2 k_2$$

$$\iff m_2 k_1 - c k_1 > m_2 k_2$$

$$\iff \frac{m_2(k_1 - k_2)}{k_1} > c$$

$$\iff \frac{k_1 - k_2}{k_1} > \frac{m_2 - m_1}{m_2}$$

□

Proof of Corollary 0.8. Equations 13 and 14 imply that

$$k^T_m = m \left\lfloor \frac{\overline{X}_m - n_m \widetilde{X}}{X^{bau} - \widetilde{X}} \right\rfloor$$

which we want to compare with equation 15. This comparison of these two values can be boiled down to comparing

$$\lfloor ab \rfloor \text{ with } \lfloor a \rfloor b$$

for some positive value a and positive integer b. Using Hermite's identity, it is apparent that

$$\lfloor ab \rfloor \geq \lfloor a \rfloor b$$

which means that

$$k_1^T \geq k_m^T \qquad \forall m \in \mathbb{N}$$

because $\lfloor m\widetilde{k}_m \rfloor = \left\lfloor m\frac{\widetilde{k}_1}{m} \right\rfloor = \lfloor \widetilde{k}_1 \rfloor$. $\square$